

2017

PHYSICS

[ Core ]

Paper — I

Full Marks : 60

Time : 3 hours

Answer all questions

*The figures in the right-hand margin indicate marks*

GROUP—A

1. (a) Find the volume of the parallelepiped whose edges are represented by : 3

$$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k},$$

$$\vec{B} = \hat{i} + 2\hat{j} - \hat{k} \text{ and}$$

$$\vec{C} = \hat{i} - \hat{j} + \hat{k}$$

- (b) If  $u$  and  $v$  are functions of  $x$  and  $y$  defined by : 3

$$x = u + e^{-v} \sin u$$

$$y = v + e^{-u} \cos u$$

( Turn Over )

( 2 )

Prove that  $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

(e) Prove that :  $x\delta(x) = 0$  3

(d) Express  $\vec{\nabla}$  in cylindrical coordinates. 3

### GROUP-B

2. (a) Prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0 \quad 4$$

(b) Prove that

$$\nabla \cdot (\nabla \phi) = \nabla^2 \phi \quad 4$$

(c) If  $u = \cos\left(\frac{xy + yz + zx}{x^2 + y^2 + z^2}\right)$ , prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0 \quad 4$$

Or

(d) If  $\phi = 3x^2y - y^3z^2$ , find grad  $\phi$  at the point  $(1, -2, -1)$ . 6

(e) The gravitational force on a particle of mass  $m$  is,  $F = -mg\hat{j}$ , find  $\nabla \times F$ . 6

3. (a) Find the minimum value of  $x^2 + y^2 + z^2$  subject to the condition that :

$$xyz = a^3. \quad 6$$

(b) Solve the equation

$$\frac{\delta y}{\delta x} - y = xy^2 \quad 6$$

Or

(c) Define Dirac Delta function. Then prove that : 6

$$\delta(x^2 - a^2) = \frac{1}{2a} [\delta(x+a) + \delta(x-a)]; a > 0$$

(d) If  $u = (1 - 2xy + y^2)$ , prove that

$$x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = y^2 u^3 \quad 6$$

4. Express curl of a vector in :

- (a) Cylindrical Coordinate System. 6
- (b) Spherical Polar Coordinate System. 6

Or

Derive expressions for velocity and acceleration in Cartesian, Cylindrical and Spherical Polar coordinates in three dimensions. 12

5. (a) State and prove Gauss Divergence theorem. 8
- (b) Prove that

$$\iiint \frac{dv}{r^2} = \iint \frac{r \cdot \hat{n}}{r^2} ds,$$

where  $\vec{r}$  is the position vector. 4

Or

- (c) Derive the equation of continuity using the Gauss divergence theorem. 6



( 5 )

(d) Evaluate the integral

$$\iiint (x^2 + y^2 + z^2) dx dy dz$$

taken over the volume enclosed by the sphere  
 $x^2 + y^2 + z^2 = 1$ .

6

**Total Pages—3**

**FSS—PHY(Core-II)(Reg)**

**2017**

**PHYSICS**

**[ Core ]**

**Paper — II**

**Full Marks : 60**

**Time : 3 hours**

**Answer all questions**

***The figures in the right-hand margin indicate marks***

**GROUP — A**

**1. Answer any *three* of the following : 4 × 3**

~~(a)~~ Conservation of angular momentum.

~~(b)~~ Poisson's ratio.

(c) Stoke's Law.

~~(d)~~ Geosynchronous orbit.

(e) Relativistic mass.

**( Turn Over )**

( 2 )

**GROUP – B**

2. Calculate moment of inertia of a solid cylinder about an axis passing through its centre and perpendicular to its length.

12

*Or*

What do you mean by fictitious force ? Give an example. Explain the weightlessness in a non-inertial frame.

4 + 2 + 6

3. Derive the expression for couple to produce unit twist of a cylindrical wire fixed at one end. Prove that a hollow cylinder is stronger than the solid cylinder.

8 + 4

*Or*

Derive Poiseuille's formula for the rate of flow of liquid through a capillary tube and discuss its limitations.

10 + 2

4. Give an expression for potential and attraction

( 3 )

due to a solid sphere at any point (a) outside  
(b) inside it. Represent the same graphically. 9 + 3

*Or*

✓ What are central forces ? Prove that angular momentum conserved under the central force. Show how the planetary motion lead to law of constant areal velocity. 2 + 5 + 5

5. Define forced vibrations. Give the theory of forced vibration and resonance. 2 + 8 + 2

*Or*

✓ Give the postulates of the special theory of relativity. Deduce Lorentz transformation equations. 12



**Total Pages—5**

**FSS—MATH(GE-I) (Reg)**

**2017**

**MATHEMATICS**

**( Calculus and Ordinary Differential Equation )**

**[ Generic Elective ]**

**Paper — I**

**Full Marks : 80**

**Time : 3 hours**

**Answer all questions**

***The figures in the right-hand margin indicate marks***

**Symbols used have their usual meaning**

**GROUP — A**

**1. Answer all questions :**

**4 × 4**

**(a) Find the radius of curvature for the curve**  
 **$p^2 = ar.$**

**( Turn Over )**

( 2 )

(b) Find the centre and radii of the sphere :

$$2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0.$$

(c) Examine the continuity of the function

$$f(x, y) = \begin{cases} xy, & \text{if } |x| \geq |y| \\ -xy, & \text{if } |x| < |y|. \end{cases}$$

at the origin.

(d) Test the exactness of the equation :

$$xdx + ydy + \frac{xdy - ydx}{(x^2 + y^2)} = 0.$$

### GROUP - B

#### [ Long Questions ]

② (a) Find the length of the one arc of the cycloid : 8

$$x = a(\theta - \sin\theta), \quad y = a(1 - \cos\theta).$$

(b) Find the Asymptote of the curve : 8

$$x^3 + y^3 - 3axy = 0.$$

( 3 )

Or

- (c) Find the radius of curvature of the curve  $y = e^x$  at the point where it crosses the y-axis. 8
- (d) Find the volume obtained by revolving the area of the parabola  $y^2 = 4ax$  lying between the vertex and the latus rectum about x-axis. 8
3. (a) Find the equation of sphere through 4 points  $(4, -1, 2), (0, -2, 3), (1, -5, -1), (2, 0, 1)$ . 8
- (b) Find the equation of the cone with vertex  $(5, 4, 3)$  and  $3x^2 + 2y^2 = 6, y + z = 0$  as base. 8

Or

- (c) Find the equation of a cylinder whose generating lines have the direction cosines  $(l, m, n)$  and which passes through the circle

$$x^2 + z^2 = a^2, y = 0. \quad 8$$

- (d) Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$  and the point  $(1, 2, 3)$ . 8

4. (a) Verify Euler's theorem :

8

$$Z = \frac{x^{1/5} + y^{1/5}}{x^2 + y^2}.$$

(b) If  $x = r \sin\theta \cdot \cos\phi$   
 $y = r \sin\theta \cdot \sin\phi$   
 $z = r \cos\theta.$

then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta.$$

8

Or

(c) Show that the function  $x^4 + x^2y + y^2$  has a minimum at  $(0, 0)$ .

8

(d) Let  $f(x, y) = \frac{x+y}{x-y},$

then discuss the existence of repeated limits and simultaneous limit at  $(0, 0)$ .

8

5. (a) Solve the initial value problem : 8

$$(2x + e^x \sin y) dx + e^x \cos y dy = 0, y(0) = \frac{\pi}{2}.$$

(b) Solve : 8

$$(D^2 + 4)y = \sin 2x.$$

Or

(c) Find particular solution of the differential equation  $(D^2 + 2D + 1)y = e^{-x} \log x$  by the method of variation of parameter. 8

(d) Solve : 8

$$y dx + (xy^2 + x - y) dy = 0.$$

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**Total Pages—2**

**FSS—ODI (AECC-I)(Reg)**

**2017**

**ODIA**

**( Elective )**

**[AECC]**

**Paper — I**

**Full Marks : 40**

**Time : 2 hours**

**Answer all questions**

*The figures in the right-hand margin indicate marks*

**କ—ବିଭାଗ**

**ସମସ୍ତ ସଂକ୍ଷିପ୍ତ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :**

**୪ × ୪**

**୧। ଯୋଗାଯୋଗର ପରିଭାଷା ଓ ଭିତ୍ତି କହିଲେ କ'ଣ ବୁଝ ?**

**୨। ଯୋଗାଯୋଗରେ କଥିତ ଓ ଲିଖିତ ଭାଷାର ଭୂମିକା କ'ଣ ?**

**( Turn Over )**



( 2 )

୩। ବ୍ୟାବସାୟିକ ଓ ସାହିତ୍ୟିକ ଯୋଗାଯୋଗର ପାର୍ଥକ୍ୟ ସଂକ୍ଷିପ୍ତରେ ବୁଝାଅ ।

୪। ସଫଳ ଯୋଗାଯୋଗରେ ଭାଷା କିପରି ହେବା ଉଚିତ ?

ଉ—ବିଭାଗ

ବୀର୍ଣ୍ଣ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

୧୨ x ୨

୫। ଯୋଗାଯୋଗର ପରିସର ଓ ଉପଯୋଗିତା ସମ୍ପର୍କରେ ଆଲୋଚନା କର ।

ଜିମ୍ବା

✓ ଯୋଗାଯୋଗର ପ୍ରକାରଭେଦ ସମ୍ପର୍କରେ ଆଲୋଚନା କର ।

୬। ଯୋଗାଯୋଗରେ ସାହିତ୍ୟିକ ଗୁଣିକା ନିର୍ଣ୍ଣୟ କର ।

ଜିମ୍ବା

✓ ଯୋଗାଯୋଗର ବାଧକ ଓ ସଫଳ ସାଧନାର ଦିଗରୁ ଦିଗରୁ ଆଲୋଚନା କର ।